1. Let $A = (1, -1), B = (-2, 0), C = (1, 2)$ and D be the vertices of a parallelogram in the $X - Y$ plane listed clockwise. Then the point D is

(A) $(4, 1)$ (B) $(-2, -3)$ (C) $(3, 0)$ (D) $(-2, 1)$

2. Let $z = (1-t^2)+i\sqrt{1-t^2}$ be a complex number where t is a real number such that $|t| < 1$. Then the locus of z in the complex plane is

(A) an ellipse (B) a hyperbola

(C) a parabola (D) a pair of straight lines

3. Let $\int_1^2 e^{x^2} dx = a$. Then the value of $\int_e^{e^4} \sqrt{\log_e x}\, dx$ is

(A) $e^4 - a$ (B) $2e^4 - a$

(C) $e^4 - e - 4a$ (D) $2e^4 - e - a$

4. The area bounded by the curves $y = e^x, y = xe^x$ and the y-axis is

(A) $e - 2$ (B) $e + 2$ (C) $e - 1$ (D) $2e - 3$

5. The set of all solutions of the inequality

$$\frac{1}{2^x - 1} > \frac{1}{1 - 2^{x-1}}$$

is

(A) $\Big(1, \infty\Big)$ (B) $\Big(0, \log_2(\frac{4}{3})\Big)$

(C) $\Big(0, \log_2(\frac{4}{3})\Big) \cup \Big(1, \infty\Big)$ (D) $\Big(-1, \infty\Big)$

6. If $\lim_{x \to 0} \frac{ae^x - b\cos x}{x} = 5$, then

(A) a and b are uniquely determined

(B) a is uniquely determined, but not b

(C) b is uniquely determined, but not a

(D) neither a nor b is uniquely determined

7. Consider four events P, Q, R and S such that if any of P and Q occurs, then either R occurs or S doesn't occur. If exactly one of R and S always occurs, which of the following statements is necessarily true? (The notation E^c denotes the complement of the event E)

(A) $R \implies P$

(B) $R \implies P^c$

(C) $R^c \implies Q^c$

(D) $R^c \implies Q$

8. The particular solution of

$$\log_e\left(\frac{dy}{dx}\right) = 5x + 7y, \quad y(0) = 0$$

is

(A) $e^{5x} + 5e^{-7y} = 7$ (B) $7e^{5x} - 5e^{-7y} = 5$

(C) $5e^{5x} + 7e^{7y} = 12$ (D) $7e^{5x} + 5e^{-7y} = 12$

9. Define $A_j = \sum_{i=1}^{n} i^j$, $j = 0, 1, 2, 3$. Then

$$\lim_{n\to\infty} \frac{A_1 A_2}{A_0 A_3}$$

is

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) 1

10. Let $p, q, r \in \mathbb{R}$. If $f(x) = px^2 + qx + r$ be such that $p + q + r = 3$ and $f(x + y) = f(x) + f(y) + xy$, for all $x, y \in \mathbb{R}$. Then the value of $f(5)$ is

(A) 25 (B) 30 (C) 35 (D) 40

11. If ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$ denote the binomial coefficients in the expansion of $(1 + x)^n$, $p > 0$ is a real number and $q = 1 - p$, then

$$\sum_{r=0}^{n} r^2 \; {}^nC_r p^{n-r} q^r$$

is

(A) np^2q^2 (B) $n^2p^2q^2$ (C) $npq + n^2p^2$ (D) $npq + n^2q^2$

12. If $|z+3-2i| = 8$ and the maximum and the minimum values of $|2z+9-8i|$ are α and β, respectively, then the value of $\alpha + \beta$ is

(A) 10 (B) 21 (C) 32 (D) 27

13. Consider the cubic equation $x^3 = 2x + 5$. Which of the following statements about the above equation is true?

(A) All its roots are real and positive
(B) It has two positive real roots and one negative real root
(C) It has two negative real roots and one positive real root
(D) It has one real root and a pair of complex roots

14. Consider two real-valued sequences $\{x_n\}$ and $\{y_n\}$ satisfying the condition $x_n^3 - y_n^3 \to 0$ as $n \to \infty$. Then, as $n \to \infty$,

(A) $x_n - y_n \to 0$ always
(B) $x_n - y_n \to 0$ only if $\{x_n\}$ converges
(C) $x_n - y_n \to 0$ only if $\{|x_n| - |y_n|\}$ converges
(D) $x_n - y_n \to 0$ only if $\{|x_n^2 + x_n y_n + y_n^2|\}$ converges

15. Let $\frac{d}{dx}P(x) = \frac{e^{\sin x}}{x}, x > 0$. If $\int_1^2 \frac{3}{x}\, e^{\sin x^3} dx = P(k) - P(1)$, then which of the following is a possible value of k?

(A) 2 (B) 4 (C) 8 (D) 16

16. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 11$ measured along a line parallel to $\frac{x}{2} = \frac{y}{3} = \frac{z}{6}$ is

(A) 5 (B) 6 (C) 7 (D) 8

17. The number of words that can be constructed using 10 letters of the English alphabet such that all five vowels appear exactly once in the word is

(A) ${}^{21}C_5\ 10!$
(B) ${}^{21}C_5\ (5!)^2$
(C) ${}^{10}P_5\ {}^{21}P_5$
(D) ${}^{10}P_5\ (21)^5$

18. Let $f : [0, \infty) \to \mathbb{R}$ be a differentiable function with $f(0) = 1$ and $f(x)f'(x) > 0$, for all x. Let $A(n)$ be the area of the region bounded by x-axis, y-axis, graph of f and the line $x = n$. Then

(A) $\{A(n)\}_{n\geq 1}$ is a convergent sequence
(B) $\{A(n)\}_{n\geq 1}$ is an oscillatory sequence
(C) the function $A : \mathbb{N} \to \mathbb{R}$ is increasing
(D) none of the above statements is true

19. Let x, y, z be three natural numbers. Then the number of triplets (x, y, z) such that $xyz = 100$ is

(A) 36 (B) 25 (C) 72 (D) 18

20. How many distinct straight lines can one form that are given by an equation $ax + by = 0$, where a and b are numbers from the set $\{0, 1, 2, 3, 4, 5, 6, 7\}$?

(A) 63 (B) 57 (C) 37 (D) 49

21. Consider three non-zero matrices A, B and C such that $ABB' = CBB'$ where B' is the transpose of B. Which of the following statements is necessarily true?

(A) $r(A) = r(C)$
(B) non-zero eigenvalues of A and C are identical
(C) $AB = CB$
(D) none of the above

22. Let m and n be nonzero integers. Define

$$A_{m,n} = \Big\{x \in \mathbb{R} : n^2x^3 + 2020x^2 + mx = 0\Big\}.$$

Then the number of pairs (m, n) for which $A_{m,n}$ has exactly two points is

(A) 0 (B) 10 (C) 16 (D) ∞

23. Consider two independent events with the same probability p $(0 < p < 1)$. Then the probability of occurrence of at least one of the two events is

(A) the same for all p
(B) linearly increasing in p
(C) strictly convex in p
(D) strictly concave in p

24. Let S be the set of all 3×3 real matrices $A = ((a_{ij}))$ such that the matrix $((a_{ij}^3))$ has rank one. Define a set $R = \{\text{rank}(A) : A \in S\}$. Then R is equal to

(A) $\{1\}$ (B) $\{1, 2\}$ (C) $\{1, 3\}$ (D) $\{1, 2, 3\}$

25. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} e^{-\frac{1}{x}}, & x > 0 \\ 0, & x \leq 0\ . \end{cases}$$

Then

(A) f is not continuous

(B) f is continuous, but not differentiable everywhere

(C) f is differentiable but f' is not continuous

(D) f is differentiable and f' is continuous

26. For a cyclic group G of order 12, the number of subgroups of G is

(A) 2 (B) 6 (C) 8 (D) 11

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuously differentiable function and $f(1) = 4$. Then the value of

$$\lim_{x \to 1} \int_4^{f(x)} \frac{2t}{x-1} dt$$

is

(A) $8f'(1)$ (B) $2f'(1)$ (C) $4f'(1)$ (D) $f'(1)$

28. The series

$$\frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} + \dots$$

(A) is uniformly convergent for all x

(B) is convergent for all x, but the convergence is not uniform

(C) is convergent only for $|x| \leq \frac{1}{2}$, but the convergence is not uniform

(D) is uniformly convergent on $[-\frac{1}{2}, \frac{1}{2}]$

29. Let S and T be two non-empty sets and $f : S \to T$ be a function such that $f(A \cap B) = f(A) \cap f(B)$ for all subsets A and B of S. Then

(A) there exist $A \subset S$ such that $f^{-1}f(A) \neq A$

(B) f is one-to-one

(C) there exist disjoint subsets A, B of S such that $f(A) \cap f(B) \neq \phi$

(D) none of the above statements is necessarily true

30. Let $x_1, x_2, ..., x_n \in \mathbb{R}$ be distinct reals. Define the set

$$A = \Big\{ \big(f_1(t), f_2(t), ..., f_n(t)\big) : t \in \mathbb{R} \Big\},$$

where for $1 \leq k \leq n$,

$$f_k(t) = \begin{cases} 1, & \text{if } \ x_k \leq t \\ 0, & \text{otherwise.} \end{cases}$$

Then A contains

(A) exactly n distinct elements
(B) exactly $(n+1)$ distinct elements
(C) exactly 2^n distinct elements
(D) infinitely many distinct elements