DU MA Economics 2020

Topic:- ECO MA S2

1) Consider independently and identically distributed random variables $X_1, . . . , X_n$ with values in [0, 2]. Each of these random variables is uniformly distributed on [0, 2]. If Y = max{$X_1, . . . , X_n$}, then the mean of Y is
[Question ID = 5844]

1. $[n/(n + 1)]^2$ [Option ID = 23370]
2. n/2(n + 1) [Option ID = 23371]
3. 2n/(n + 1) [Option ID = 23372]
4. n/(n + 1) [Option ID = 23373]

Correct Answer :-

- 2n/(n + 1) [Option ID = 23372]

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2) A coin toss has possible outcomes H and T with probabilities 3/4 and 1/4 respectively. A gambler observes a sequence of tosses of this coin until H occurs. Let the first H occur on the n^{th} toss. If n is odd, then the gambler's prize is -2^n, and if n is even, then the gambler's prize is 2^n. What is the expected value of the gambler's prize?
[Question ID = 5845]

1. 1 [Option ID = 23374]
2. - 1 [Option ID = 23375]
3. 3 [Option ID = 23376]
4. - 3 [Option ID = 23377]

Correct Answer :-

- - 1 [Option ID = 23375]

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3) Suppose two fair dice are tossed simultaneously. What is the probability that the total number of spots on the upper faces of the two dice is not divisible by 2, 3, or 5?
[Question ID = 5846]

1. 1/3 [Option ID = 23378]
2. 2/9 [Option ID = 23379]
3. 4/9 [Option ID = 23380]
4. 7/16 [Option ID = 23381]

Correct Answer :-

- 2/9 [Option ID = 23379]

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4) A student is answering a multiple-choice examination. Suppose a question has m possible answers. The student knows the correct answer with probability p. If the student knows the correct answer, then she picks that answer; otherwise, she picks randomly from the choices with probability 1/m each. Given that the student picked the correct answer, the probability that she knew the correct answer is
[Question ID = 5847]

1. mp/[1 + (m − 1)p] [Option ID = 23382]
2. mp/[1 + (1 − p)m] [Option ID = 23383]
3. (1 − p)/[1 + (m − 1)p] [Option ID = 23384]
4. (1 − p)/[1 + (1 − p)m] [Option ID = 23385]

Correct Answer :-

- mp/[1 + (m − 1)p] [Option ID = 23382]

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5) Suppose Y is a random variable with uniform distribution on [0, 2]. The value of the cumulative distribution function of the random variable $X = e^Y$ at $x \in [1, e^2]$ is
[Question ID = 5848]

1. $2^{-1} \ln x$ [Option ID = 23386]
2. $4^{-1} \ln x - 2^{-1}$ [Option ID = 23387]
3. $\ln x$ [Option ID = 23388]
4. $\ln x - 1$ [Option ID = 23389]

Correct Answer :-

- $2^{-1} \ln x$ [Option ID = 23386]

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6) Consider an economy where the final commodity is produced by a single firm using labour only. The price-setting firm charges a 25% mark-up over its per unit nominal wage cost. The workers demand a real wage rate W/P = (1 − u), where u is the unemployment rate, P is the price, and W is the nominal wage rate. The natural rate of unemployment in this economy is
[Question ID = 5849]

1. 20% [Option ID = 23390]
2. 17% [Option ID = 23391]

3. 13% [Option ID = 23392]
4. 10% [Option ID = 23393]

Correct Answer :-

- 20% [Option ID = 23390]

7) **The aggregate production function of an economy is $Y_t = (K_t L_t)^{1/2}$. Capital grows according to $K_{t+1} = (1 - \delta) K_t + S_t$, where $S_t = sY_t$, $L_t = \bar{L}$, s is the saving rate, δ is the depreciation rate and $\bar{L}$ is the total population. Then, the steady-state level of consumption per capita is**

[Question ID = 5850]

1. s/δ

   [Option ID = 23394]
2. s^2/δ^2

   [Option ID = 23395]
3. $\delta^{1/s}$

   [Option ID = 23396]
4. $s(1 - s)/\delta$

   [Option ID = 23397]

Correct Answer :-

- $s(1 - s)/\delta$

  [Option ID = 23397]

8) **Consider a production technology Y = AL, where Y is output, A is productivity, and L is labour input. A firm sets its price P at a constant mark-up μ over the effective wage cost per unit of production W/A. The expected real wage rate of workers is $W/P^e = A^{\alpha}(1 - u)^{1-\alpha}$, where $0 < \alpha < 1$ and P^e is the expected price. If the price expected by workers matches the actual price level, then the effect of a rise in the level of productivity on unemployment is**

[Question ID = 5851]

1. positive

   [Option ID = 23398]
2. negative

   [Option ID = 23399]
3. zero

   [Option ID = 23400]
4. ambiguous

   [Option ID = 23401]

Correct Answer :-

- negative

  [Option ID = 23399]

9) **A household has an endowment of 1 unit of time. The household maximises its utility u = ln c + b ln(1 − l), where c denotes consumption and $l \in [0, 1]$ denotes time spent working. It finances its consumption from labour income wl, where w is the market wage rate per unit of labour time. If the market wage rate goes up, then equilibrium labour supply of the household**

[Question ID = 5852]

1. increases

   [Option ID = 23402]
2. decreases

   [Option ID = 23403]
3. remains constant

   [Option ID = 23404]
4. changes in an ambiguous manner

   [Option ID = 23405]

Correct Answer :-

- remains constant

  [Option ID = 23404]

10) **Consider the IS-LM model with a given price level P. Investment is a decreasing function of the interest rate and savings is an increasing function of aggregate income. The demand for real money balances M/P is an increasing function of aggregate income and a decreasing function of the interest rate. The monetary authority chooses nominal money supply M**

to ensure that the resulting money market equilibrium keeps the interest rate fixed at some target level. In this setup, an increase in the target interest rate leads to

[Question ID = 5853]

1. a rise in equilibrium output [Option ID = 23406]
2. a fall in equilibrium output [Option ID = 23407]
3. no effect on equilibrium output [Option ID = 23408]
4. an ambiguous effect on equilibrium output [Option ID = 23409]

Correct Answer :-

- a fall in equilibrium output [Option ID = 23407]

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11) Consider the Solow growth model with a given savings ratio, a constant population growth rate, zero rate of capital depreciation, and no technical progress. Let k* be the steady-state capital-labour ratio in this economy. Suppose the economy is yet to reach the steady-state and has capital-labour ratio k_1 at time t_1 and capital-labour ratio k_2 at time t_2, such that $t_1 < t_2$ and $k_1 < k_2 < k^*$. Let the associated growth rates of *per* capita income at time t_1 and t_2 be g_1 and g_2 respectively. Then, by the properties of the Solow model,

[Question ID = 5854]

1. $g_1 < g_2$

   [Option ID = 23410]
2. $g_1 > g_2$

   [Option ID = 23411]
3. $g_1 = g_2$

   [Option ID = 23412]
4. the relationship between g_1 and g_2 is ambiguous

   [Option ID = 23413]

Correct Answer :-

- $g_1 > g_2$

  [Option ID = 23411]

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12) A consumer lives for periods 1 and 2. Given consumptions c_1 and c_2 in these periods, her utility is $U = \ln c_1 + (1 + \rho)^{-1} \ln c_2$. She earns incomes w_1 and w_2 in the two periods and her lifetime budget constraint is $c_1 + (1 + r)^{-1}c_2 = w_1 + (1 + r)^{-1}w_2$, where r is the interest rate on savings. If $r > \rho$, then

[Question ID = 5855]

1. $c_1 > c_2$

   [Option ID = 23414]
2. $c_1 < c_2$

   [Option ID = 23415]
3. $c_1 = c_2$

   [Option ID = 23416]
4. The relationship between c_1 and c_2 is ambiguous

   [Option ID = 23417]

Correct Answer :-

- $c_1 < c_2$

  [Option ID = 23415]

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13) A consumer lives for periods 1 and 2. Her lifetime utility function is $U(c_1, c_2) = (c_1^\gamma + c_2^\gamma)/\gamma$ where $0 < \gamma < 1$ and c_i is consumption in period i. The elasticity of substitution between consumption in period 1 and consumption in period 2 is

[Question ID = 5856]

1. $1 + \gamma$

   [Option ID = 23418]
2. $1 - \gamma$

   [Option ID = 23419]
3. $1/(1 + \gamma)$

   [Option ID = 23420]
4. $1/(1 - \gamma)$

   [Option ID = 23421]

Correct Answer :-

- $1/(1 - \gamma)$

  [Option ID = 23421]

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14) A and B play a best-of-seven table-tennis match, i.e., the first to win four games will win the match. The two players are equally likely to win any of the games in the match. The probability that the match will end in 6 games is

[Question ID = 5857]

1. less than the probability that it will end in 7 games

   [Option ID = 23422]
2. equal to the probability that it will end in 7 games

   [Option ID = 23423]
3. greater than the probability that it will end in 7 games

   [Option ID = 23424]
4. None of these

   [Option ID = 23425]

Correct Answer :-

- equal to the probability that it will end in 7 games

  [Option ID = 23423]

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15) Let X and Y be jointly normally distributed, i.e., $(X,Y) \sim N(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$

If $\sigma_X^2 = \sigma_Y^2$ and $0 < \rho < 1$, then

[Question ID = 5858]

1. the OLS regression of Y on X will yield a slope that is less than unity, and that of X on Y will yield a slope greater than unity

   [Option ID = 23426]
2. the OLS regression of Y on X will yield a slope that is less than unity, and that of X on Y will yield a slope less than unity

   [Option ID = 23427]
3. the OLS regression of Y on X will yield a slope that is greater than unity, and that of X on Y will yield a slope less than unity

   [Option ID = 23428]
4. it is not possible to draw conclusions about the magnitude of the slope with the given information

   [Option ID = 23429]

Correct Answer :-

- the OLS regression of Y on X will yield a slope that is less than unity, and that of X on Y will yield a slope less than unity

  [Option ID = 23427]

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16) Let ¬ denote the negation of a statement. Consider a set X and a binary relation ≻ on X. Relation ≻ is said to be irreflexive if ¬x ≻ x for every x ∈ X. Relation ≻ is said to be transitive if, for all x, y, z ∈ X, x ≻ y and y ≻ z implies x ≻ z.

If ≻ is asymmetric (i.e., for all x, y ∈ X, x ≻ y implies ¬y ≻ x) and negatively transitive (i.e., for all x, y, z ∈ X, x ≻ y implies x ≻ z, or z ≻ y, or both), then ≻ is

[Question ID = 5859]

1. irreflexive, but not transitive

   [Option ID = 23430]
2. transitive, but not irreflexive

   [Option ID = 23431]
3. irreflexive and transitive

   [Option ID = 23432]
4. neither transitive, nor irreflexive

   [Option ID = 23433]

Correct Answer :-

- irreflexive and transitive

  [Option ID = 23432]

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17) Let ¬ denote the negation of a statement. Consider a set X and a binary relation ≻ on X. For all x, y ∈ X, we say x ≽ y if and only if ¬y ≻ x. Relation ≽ is said to be total if, for all x, y ∈ X, ¬x ≽ y implies y ≽ x.

If ≻ is asymmetric (i.e., for all x, y ∈ X, x ≻ y implies ¬y ≻ x) and negatively transitive (i.e., for all x, y, z ∈ X, x ≻ y implies x ≻ z, or z ≻ y, or both), then ≽

[Question ID = 5860]

1. is not total

   [Option ID = 23434]
2. is total

[Option ID = 23435]
3. may not be total

[Option ID = 23436]
4. is not total over a nonempty subset of X

[Option ID = 23437]

Correct Answer :-
- is total

[Option ID = 23435]

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18) Let $\neg$ denote the negation of a statement. Consider a set X and a binary relation $\succ$ on X. For all $x, y \in X$, we say $x \succcurlyeq y$ if and only if $\neg y \succ x$. Relation $\succcurlyeq$ is said to be transitive if, for all $x, y, z \in X$, $x \succcurlyeq y$ and $y \succcurlyeq z$ implies $x \succcurlyeq z$.

If $\succ$ is asymmetric (i.e., for all $x, y \in X$, $x \succ y$ implies $\neg y \succ x$) and negatively transitive (i.e., for all $x, y, z \in X$, $x \succ y$ implies $x \succ z$, or $z \succ y$, or both), then $\succcurlyeq$

[Question ID = 5861]
1. is not transitive over a nonempty subset of X

[Option ID = 23438]
2. is not transitive

[Option ID = 23439]
3. may not be transitive

[Option ID = 23440]
4. is transitive

[Option ID = 23441]

Correct Answer :-
- is transitive

[Option ID = 23441]

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19) Let $\neg$ denote the negation of a statement. Consider a set X and a binary relation $\succ$ on X. For all $x, y \in X$, we say $x \sim y$ if and only if $\neg x \succ y$ and $\neg y \succ x$. Relation $\sim$ is said to be transitive if, for all $x, y, z \in X$, $x \sim y$ and $y \sim z$ implies $x \sim z$.

If $\succ$ is asymmetric (i.e., for all $x, y \in X$, $x \succ y$ implies $\neg y \succ x$) and negatively transitive (i.e., for all $x, y, z \in X$, $x \succ y$ implies $x \succ z$, or $z \succ y$, or both), then $\sim$

[Question ID = 5862]
1. is transitive

[Option ID = 23442]
2. is not transitive

[Option ID = 23443]
3. may not be transitive

[Option ID = 23444]
4. is not transitive over a nonempty subset of X

[Option ID = 23445]

Correct Answer :-
- is transitive

[Option ID = 23442]

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20) Let $\neg$ denote the negation of a statement. Consider a set X and a binary relation $\succ$ on X. For all $x, y \in X$, we say $x \sim y$ if and only if $\neg x \succ y$ and $\neg y \succ x$. Relation $\sim$ is said to be symmetric if, for all $x, y \in X$, $x \sim y$ implies $y \sim x$.

If $\succ$ is asymmetric (i.e., for all $x, y \in X$, $x \succ y$ implies $\neg y \succ x$) and negatively transitive (i.e., for all $x, y, z \in X$, $x \succ y$ implies $x \succ z$, or $z \succ y$, or both), then $\sim$

[Question ID = 5863]
1. is symmetric

[Option ID = 23446]
2. is not symmetric

[Option ID = 23447]
3. may not be symmetric

[Option ID = 23448]
4. is not symmetric over a nonempty subset of X

[Option ID = 23449]

Correct Answer :-

- is symmetric

  [Option ID = 23446]

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21) Consider the following game for players 1 and 2. Player 1 moves first and chooses L or R. If she chooses L, then the game ends and the payoffs are (1, 0), where the first entry is 1's payoff and the second entry is 2's payoff. If she chooses R, then 2 chooses U or D. If she chooses U, then the game ends and the payoffs are (0, 2). If she chooses D, then 1 chooses L or R. If she chooses L, then the game ends and the payoffs are (4, 0). If she chooses R, then the game ends and the payoffs are (3, 3). This game has
[Question ID = 5864]

1. one Nash equilibrium in pure strategies [Option ID = 23450]
2. two Nash equilibria in pure strategies [Option ID = 23451]
3. three Nash equilibria in pure strategies [Option ID = 23452]
4. no Nash equilibria in pure strategies [Option ID = 23453]

Correct Answer :-

- two Nash equilibria in pure strategies [Option ID = 23451]

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22) Consider the following game for players 1 and 2. Player 1 moves first and chooses L or R. If she chooses L, then the game ends and the payoffs are (1, 0), where the first entry is 1's payoff and the second entry is 2's payoff. If she chooses R, then 2 chooses U or D. If she chooses U, then the game ends and the payoffs are (0, 2). If she chooses D, then 1 chooses L or R. If she chooses L, then the game ends and the payoffs are (4, 0). If she chooses R, then the game ends and the payoffs are (3, 3). This game has
[Question ID = 5865]

1. one subgame perfect Nash equilibrium [Option ID = 23454]
2. two subgame perfect Nash equilibria [Option ID = 23455]
3. three subgame perfect Nash equilibria [Option ID = 23456]
4. no subgame perfect Nash equilibria [Option ID = 23457]

Correct Answer :-

- one subgame perfect Nash equilibrium [Option ID = 23454]

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23) In a non-cooperative game, if a profile of strategies
[Question ID = 5866]

1. is a Nash equilibrium, then it is an equilibrium in dominant strategies [Option ID = 23458]
2. is a Nash equilibrium, then it is a subgame perfect equilibrium [Option ID = 23459]
3. is a Nash equilibrium, then it is a sequential equilibrium [Option ID = 23460]
4. is an equilibrium in dominant strategies, then it is a Nash equilibrium [Option ID = 23461]

Correct Answer :-

- is an equilibrium in dominant strategies, then it is a Nash equilibrium [Option ID = 23461]

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24) If player 1 is the row player and player 2 is the column player in games

$$G := \begin{array}{c} \\ \mathrm{U} \\ \mathrm{D} \end{array} \overset{\begin{array}{cc} \mathrm{L} & \mathrm{R} \end{array}}{\begin{pmatrix} a,b & c,d \\ e,f & g,h \end{pmatrix}} \quad \text{and} \quad G' := \begin{array}{c} \\ \mathrm{U} \\ \mathrm{D} \end{array} \overset{\begin{array}{ccc} \mathrm{L} & \mathrm{M} & \mathrm{R} \end{array}}{\begin{pmatrix} a,b & \alpha,\beta & c,d \\ e,f & \gamma,\delta & g,h \end{pmatrix}}$$ **then**

[Question ID = 5867]

1. 2's payoff in a Nash equilibrium of G′ cannot be less than 2's payoff in a Nash equilibrium of G

   [Option ID = 23462]
2. 2's payoff in a Nash equilibrium of G cannot be less than 2's payoff in a Nash equilibrium of G′

   [Option ID = 23463]
3. 2's payoff in a Nash equilibrium of G must be equal to 2's payoff in a Nash equilibrium of G′

   [Option ID = 23464]
4. 2's payoff in a Nash equilibrium of G may be higher than 2's payoff in a Nash equilibrium of G′

   [Option ID = 23465]

Correct Answer :-

- 2's payoff in a Nash equilibrium of G may be higher than 2's payoff in a Nash equilibrium of G′

  [Option ID = 23465]

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25) Consider an exchange economy with agents 1 and 2 and goods *x* and *y*. Agent 1 lexicographically prefers *x* to *y*, i.e., between two non-identical bundles of *x* and *y*, she strictly prefers the bundle with more of *x*, but if the bundles have the same amount of *x*, then she strictly prefers the bundle with more of *y*.

Agent 2's utility function is $u_2(x,y) = x + y$

Agent 1's endowment is $(\omega_x^1, \omega_y^1) = (0,10)$ **and Agent 2's endowment is** $(\omega_x^2, \omega_y^2) = (10,0)$

The set of competitive equilibrium price ratios p_x/p_y **for this economy is**

[Question ID = 5868]
1. {1}

   [Option ID = 23466]
2. [0,1]

   [Option ID = 23467]
3. (0,1]

   [Option ID = 23468]
4. ∅

   [Option ID = 23469]

Correct Answer :-
- {1}

  [Option ID = 23466]

26) Consider an exchange economy with agents 1 and 2 and goods *x* and *y*.

Agent 1 lexicographically prefers *y* to *x*, i.e., between two non-identical bundles of *x* and *y*, she strictly prefers the bundle with more of *y*, but if the bundles have the same amount of *y*, then she strictly prefers the bundle with more of *x*.

Agent 2's utility function is $u_2(x,y) = x + y$

Agent 1's endowment is $(\omega_x^1, \omega_y^1) = (0,10)$ and Agent 2's endowment is $(\omega_x^2, \omega_y^2) = (10,0)$

The set of competitive equilibrium price ratios p_x/p_y for this economy is

[Question ID = 5869]
1. {1}

   [Option ID = 23470]
2. [0,1]

   [Option ID = 23471]
3. (0,1]

   [Option ID = 23472]
4. ∅

   [Option ID = 23473]

Correct Answer :-
- (0,1]

  [Option ID = 23472]

27) Consider an exchange economy with goods *x* and *y*, and agents 1 and 2, whose endowments are $(\omega_x^1, \omega_y^1) = (0,9)$ and $(\omega_x^2, \omega_y^2) = (10,0)$ respectively.

The utility functions of 1 and 2 are $u_1(x,y) = \min\{x,y\}$ and $u_2(x,y) = \min\{x,y\}$ respectively.

The competitive equilibrium price ratio p_x/p_y is

[Question ID = 5870]
1. 9/10

   [Option ID = 23474]
2. 10/9

   [Option ID = 23475]
3. 1

   [Option ID = 23476]
4. 0

   [Option ID = 23477]

Correct Answer :-
- 0

  [Option ID = 23477]

28) Consider an exchange economy with goods *x* and *y*, and agents 1 and 2, whose endowments are $(\omega_x^1, \omega_y^1) = (0,9)$ and $(\omega_x^2, \omega_y^2) = (10,0)$ respectively.

The utility functions of 1 and 2 are $u_1(x,y) = \min\{x,y\}$ and $u_2(x,y) = \min\{x,y\}$ respectively.

The competitive equilibrium allocations are

[Question ID = 5871]

1. 1 gets (10 − x, 9 − y) and 2 gets (x, y), where x ∈ [9, 10] and y = 9

   [Option ID = 23478]
2. 1 gets (x, y) and 2 gets (10 − x, 9 − y), where x ∈ [9, 10] and y = 9

   [Option ID = 23479]
3. 1 gets (x, y) and 2 gets (9 − x, 10 − y), where x ∈ [8, 9] and y = 10

   [Option ID = 23480]
4. 1 gets (x, y) and 2 gets (9 − x, 10 − y), where x = 9 and y ∈ [9, 10]

   [Option ID = 23481]

Correct Answer :-

- 1 gets (x, y) and 2 gets (10 − x, 9 − y), where x ∈ [9, 10] and y = 9

  [Option ID = 23479]

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29) Consider an exchange economy with goods *x* and y, and agents 1 and 2, whose endowments are $(\omega_x^1, \omega_y^1) = (0, 9)$ and $(\omega_x^2, \omega_y^2) = (10, 0)$ respectively.

The utility functions of 1 and 2 are $u_1(x, y) = \min\{x, y\}$ and $u_2(x, y) = \min\{x, y\}$ respectively.

The allocation that gives (10, 9) to 1 and (0,0) to 2 is

[Question ID = 5872]

1. Pareto efficient but not a competitive equilibrium allocation

   [Option ID = 23482]
2. neither Pareto efficient nor a competitive equilibrium allocation

   [Option ID = 23483]
3. a competitive equilibrium allocation that is Pareto efficient

   [Option ID = 23484]
4. a competitive equilibrium allocation that is not Pareto efficient

   [Option ID = 23485]

Correct Answer :-

- a competitive equilibrium allocation that is Pareto efficient

  [Option ID = 23484]

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30) Given a non-empty set C$\subset \mathbb{R}^n$, for every p ∈ $\mathbb{R}^n_+$, let c(p) ∈ C be such that $p.c(p) \le p.c$ for every c ∈ C. Then, the function $e: \mathbb{R}^n_+ \to \mathbb{R}$ given by $e(p) = p.c(p)$ is

[Question ID = 5873]

1. linear

   [Option ID = 23486]
2. convex

   [Option ID = 23487]
3. concave

   [Option ID = 23488]
4. quasi-convex

   [Option ID = 23489]

Correct Answer :-

- concave

  [Option ID = 23488]

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31) Given a non-empty set C$\subset \mathbb{R}^n$, for every p ∈ $\mathbb{R}^n_+$, let c(p) ∈ C be such that $p.c(p) \le p.c$ for every c ∈ C. Then, the function $e: \mathbb{R}^n_+ \to \mathbb{R}$ given by $e(p) = p.c(p)$ is

[Question ID = 5874]

1. homogenous of degree 0

   [Option ID = 23490]
2. homogenous of degree 1

   [Option ID = 23491]
3. homogenous of degree ∞

   [Option ID = 23492]
4. non-homogenous

   [Option ID = 23493]

Correct Answer :-

- homogenous of degree 1

  [Option ID = 23491]

32) **Suppose $u: \mathbb{R} \to \mathbb{R}_+$ is strictly increasing and has the supremum (i.e., least upper bound) α ∈ $\mathbb{R}$. Then, the function $x \mapsto u(x)/[\alpha - u(x)]$ is**

[Question ID = 5875]

1. not well defined for some $x \in \mathbb{R}$

   [Option ID = 23494]
2. bounded above

   [Option ID = 23495]
3. unbounded above

   [Option ID = 23496]
4. not strictly increasing

   [Option ID = 23497]

Correct Answer :-

- unbounded above

  [Option ID = 23496]

33) **The interval [0, ∞) can be expressed as**

[Question ID = 5876]

1. $\cap_{n=1}^{\infty} (a_n, \infty)$, where each a_n is a rational number

   [Option ID = 23498]
2. $\cup_{n=1}^{\infty} (a_n, b_n]$, where each a_n and b_n is a real number

   [Option ID = 23499]
3. $\cap_{n=1}^{\infty} [a_n, b_n]$, where each a_n and b_n is an irrational number

   [Option ID = 23500]
4. All of these

   [Option ID = 23501]

Correct Answer :-

- $\cap_{n=1}^{\infty} (a_n, \infty)$, where each a_n is a rational number

  [Option ID = 23498]

34) **If $f: \mathbb{R}^2 \to \mathbb{R}$ is given by**

$$f(x,y) = \begin{cases} xy/(x^2+y^2), & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$

then

[Question ID = 5877]

1. f is differentiable at (0, 0) and both partial derivatives at (0, 0) are 0

   [Option ID = 23502]
2. f is non-differentiable at (0, 0) and both partial derivatives at (0, 0) are 0

   [Option ID = 23503]
3. f is differentiable at (0, 0) and neither partial derivative at (0, 0) is 0

   [Option ID = 23504]
4. f is non-differentiable at (0, 0) and neither partial derivative at (0, 0) exists

   [Option ID = 23505]

Correct Answer :-

- f is non-differentiable at (0, 0) and both partial derivatives at (0, 0) are 0

  [Option ID = 23503]

35) **Suppose $f: \mathbb{R} \to \mathbb{R}$ is a twice-differentiable function that solves the differential equation $D^2 f - Df - f - 1 = 0$ over $\mathbb{R}$ and satisfies the condition $f(0) = 0 = f(k)$ for some $k > 0$. Then,**

[Question ID = 5878]

1. f has positive and negative values over (0, k)

   [Option ID = 23506]
2. f has only positive values over (0, k)

[Option ID = 23507]
3. f has only negative values over (0, k)

   [Option ID = 23508]
4. f = −1 on (0, k)

   [Option ID = 23509]

Correct Answer :-

- f has only negative values over (0, k)

  [Option ID = 23508]

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36) Let $\mathcal{B}$ be the collection of sets $E \subset R$ satisfying: for every $x \in E$, there are real numbers a and b such that $a < b$ and $x \in (a,b) \subset E$

Let $\mathcal{C}$ be the collection of sets $E \subset R$ satisfying: for every $x \in E$, there are rational numbers a and b such that $a < b$ and $x \in (a,b) \subset E$

then,

[Question ID = 5879]

1. $\mathcal{B} \subset \mathcal{C}$ and $\mathcal{B} \neq \mathcal{C}$

   [Option ID = 23510]
2. $\mathcal{C} \subset \mathcal{B}$ and $\mathcal{B} \neq \mathcal{C}$

   [Option ID = 23511]
3. $\mathcal{B} = \mathcal{C}$

   [Option ID = 23512]
4. Neither $\mathcal{B} \subset \mathcal{C}$ nor $\mathcal{C} \subset \mathcal{B}$

   [Option ID = 23513]

Correct Answer :-

- $\mathcal{B} = \mathcal{C}$

  [Option ID = 23512]

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37) The set $\{(x,y) \in \mathbb{R}^2 | \, x > 0$ and $y \le \text{In}\, x - e^x\}$ is

[Question ID = 5880]

1. a linear subspace of $\mathbb{R}^2$

   [Option ID = 23514]
2. convex

   [Option ID = 23515]
3. non-convex

   [Option ID = 23516]
4. a convex polytope

   [Option ID = 23517]

Correct Answer :-

- convex

  [Option ID = 23515]

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38) Suppose the distance between $x, y \in \mathbb{R}$ is given by $|x - y|$ and $f: \mathbb{R} \to \mathbb{R}$ is a continuous function. If *E* is an open subset of $\mathbb{R}$, then $\{x \in \mathbb{R} \mid f(x) \in E\}$ is

[Question ID = 5881]

1. An open subset of $\mathbb{R}$

   [Option ID = 23518]
2. A closed subset of $\mathbb{R}$

   [Option ID = 23519]
3. Neither an open, nor a closed, subset of $\mathbb{R}$

   [Option ID = 23520]
4. An open and closed subset of $\mathbb{R}$

   [Option ID = 23521]

Correct Answer :-

- An open subset of $\mathbb{R}$

[Option ID = 23518]

39) Which of the following two numbers is larger for $k \neq 0:\ e^{k\pi} or\ \pi^{ke}$?

[Question ID = 5882]

1. $e^{k\pi}$

   [Option ID = 23522]
2. π^{ke}

   [Option ID = 23523]
3. They are equal

   [Option ID = 23524]
4. It depends on the value of k

   [Option ID = 23525]

Correct Answer :-

- It depends on the value of k

  [Option ID = 23525]

40) Consider the matrix $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

where $\theta \in [0, 2\pi)$. The inner product of vectors $v = (v_1, v_2)$ and $w = (w_1, w_2)$ in $\mathbb{R}^2$ is defined by $\langle v, w \rangle = v_1 w_1 + v_2 w_2$. So, for the vectors v and w in $\mathbb{R}^2$

[Question ID = 5883]

1. $\langle Av, Aw \rangle = \langle v, w \rangle$

   [Option ID = 23526]
2. $\langle Av, Aw \rangle > \langle v, w \rangle$

   [Option ID = 23527]
3. $\langle Av, Aw \rangle < \langle v, w \rangle$

   [Option ID = 23528]
4. The comparison of $\langle Av, Aw \rangle$ and $\langle v, w \rangle$ depends on the value of θ

   [Option ID = 23529]

Correct Answer :-

- $\langle Av, Aw \rangle = \langle v, w \rangle$

  [Option ID = 23526]

41) Let $\lfloor x \rfloor$ be the greatest integer that is less than or equal to $x \in \mathbb{R}$. The function $f: \mathbb{R} \to \mathbb{R}$, defined by $f(x) = x - \lfloor x \rfloor$ for $x \in \mathbb{R}$, is

[Question ID = 5884]

1. Left-discontinuous at an integer

   [Option ID = 23530]
2. Right-discontinuous at an integer

   [Option ID = 23531]
3. Left discontinuous and right-discontinuous at an integer

   [Option ID = 23532]
4. Discontinuous everywhere

   [Option ID = 23533]

Correct Answer :-

- Left-discontinuous at an integer

  [Option ID = 23530]

42) Let $\lceil x \rceil$ be the smallest integer that is greater than or equal to $x \in \mathbb{R}$. The function $f: \mathbb{R} \to \mathbb{R}$, defined by $f(x) = \lceil x \rceil - x$ for $x \in \mathbb{R}$ is

[Question ID = 5885]

1. Left-discontinuous at an integer

   [Option ID = 23534]
2. Right-discontinuous at an integer

   [Option ID = 23535]
3. Left discontinuous and right-discontinuous at an integer

[Option ID = 23536]
4. Discontinuous everywhere

[Option ID = 23537]

Correct Answer :-
- Right-discontinuous at an integer

[Option ID = 23535]

43) Let $\lfloor x \rfloor$ be the greatest integer that is less than or equal to $x \in \mathbb{R}$. Let $\lceil x \rceil$ be the smallest integer that is greater than or equal to $x \in \mathbb{R}$. The function $f: \mathbb{R} \to \mathbb{R}$, defined by $f(x) = \lceil x \rceil - \lfloor x \rfloor$ for $x \in \mathbb{R}$, is

[Question ID = 5886]
1. Left-discontinuous at an integer

[Option ID = 23538]
2. Right-discontinuous at an integer

[Option ID = 23539]
3. Left discontinuous and right-discontinuous at an integer

[Option ID = 23540]
4. Discontinuous everywhere

[Option ID = 23541]

Correct Answer :-
- Left discontinuous and right-discontinuous at an integer

[Option ID = 23540]

44) If

$A = \{(x, y) \in \mathbb{R}^2 | \; x \geq 0, y \geq 0, xy \geq 1\}$

$B = \{(x, y) \in \mathbb{R}^2 | \; x \leq 0, y \geq 0, xy \leq -1\}$

and

$C = \{a + b | a \in A, b \in B\}$

Then,

[Question ID = 5887]
1. $\{(x, y) \in \mathbb{R}^2 | x = 0, y \geq 0\}$ is a subset of C

[Option ID = 23542]
2. $\{(x, y) \in \mathbb{R}^2 | x = 0, y > 0\}$ is a subset of C

[Option ID = 23543]
3. $\{(x, y) \in \mathbb{R}^2 | x \geq 0, y = 0\}$ is a subset of C

[Option ID = 23544]
4. $\{(x, y) \in \mathbb{R}^2 | x > 0, y = 0\}$ is a subset of C

[Option ID = 23545]

Correct Answer :-
- $\{(x, y) \in \mathbb{R}^2 | x = 0, y > 0\}$ is a subset of C

[Option ID = 23543]

45) Consider a 4 x 4-matrix A. Obtain matrix B from matrix A by performing the following operations in sequence:

(1) Interchange the first and fourth columns, and then

(2) Interchange the second and fourth rows. Then,

[Question ID = 5888]
1. det A = det B

[Option ID = 23546]
2. det A $\neq$ det B

[Option ID = 23547]
3. det B $\leq$ 0

[Option ID = 23548]
4. det B > 0

[Option ID = 23549]

Correct Answer :-

- det A = det B

  [Option ID = 23546]

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46) The maximum value of $f(x,y) = xy$, subject to $|x| \geq |y|$ and $|x| + |y| \leq 1$, is

[Question ID = 5889]

1. 1/4

   [Option ID = 23550]
2. 1/2

   [Option ID = 23551]
3. 4

   [Option ID = 23552]
4. 2

   [Option ID = 23553]

Correct Answer :-

- 1/4

  [Option ID = 23550]

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47) Consider a decreasing differentiable function $g: \mathbb{R}_+ \to \mathbb{R}_+$ and an increasing continuous function $f: \mathbb{R}_+ \to \mathbb{R}_+$. If $F: \mathbb{R}_+ \to \mathbb{R}_+$ satisfies $F(x) = \int_0^{g(x)} f(t)dt$ for every $x \in \mathbb{R}_+$, then F is

[Question ID = 5890]

1. Increasing over [0, a] and decreasing over [a, ∞), for some a > 0

   [Option ID = 23554]
2. Decreasing over [0, a] and increasing over [a, ∞), for some a > 0

   [Option ID = 23555]
3. Increasing

   [Option ID = 23556]
4. Decreasing

   [Option ID = 23557]

Correct Answer :-

- Decreasing

  [Option ID = 23557]

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48) If $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ are defined by

$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational} \end{cases}$ **and** $g(x) = \begin{cases} 0, & \text{if } x \text{ is irrational} \\ x, & \text{if } x \text{ is rational} \end{cases}$

Then $h: \mathbb{R} \to \mathbb{R}$, given by $h(x) = f(x) - g(x)$, is

[Question ID = 5891]

1. Injective but not surjective

   [Option ID = 23558]
2. Surjective but not injective

   [Option ID = 23559]
3. Neither injective nor surjective

   [Option ID = 23560]
4. Bijective

   [Option ID = 23561]

Correct Answer :-

- Bijective

  [Option ID = 23561]

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49) Given nonempty subsets of $\mathbb{R}^2$, say $Y_1, \ldots, Y_n$, let

$$Y^* = \left\{ \sum_{j=1}^{n} y_j | y_1 \in Y_1, y_2 \in Y_2, \ldots, y_n \in Y_n \right\}$$

Fix p $\in \mathbb{R}^2$. For a nonempty set X$\subset \mathbb{R}^2$, let $v(p,X) = \sup\{p.x | x \in X\}$

Suppose there exists $y^* \in Y^*$ such that $p.y^* = v(p,Y^*)$, and for every $i \in \{1,\ldots,n\}$, there exists $y_i \in Y_i$ such that $p.y_i = v(p,Y_i)$.

Then,

[Question ID = 5892]

1. $v(p,Y^*) < \sum_{j=1}^{n} v(p,Y_j)$ or $v(p,Y^*) > \sum_{j=1}^{n} v(p,Y_j)$

   [Option ID = 23562]
2. $v(p,Y^*) = \sum_{j=1}^{n} v(p,Y_j)$

   [Option ID = 23563]
3. $v(p,Y^*) < \sum_{j=1}^{n} v(p,Y_j)$

   [Option ID = 23564]
4. $v(p,Y^*) > \sum_{j=1}^{n} v(p,Y_j)$

   [Option ID = 23565]

Correct Answer :-

- $v(p,Y^*) = \sum_{j=1}^{n} v(p,Y_j)$

  [Option ID = 23563]

50) If $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 2 & -2 & 0 \\ 4 & 3 & 2 & -1 \end{bmatrix}$

and A^T is the transpose of A, then $\det(A^T A)$ is

[Question ID = 5893]

1. 16

   [Option ID = 23566]
2. -16

   [Option ID = 23567]
3. 4

   [Option ID = 23568]
4. -4

   [Option ID = 23569]

Correct Answer :-

- 16

  [Option ID = 23566]