ACTUARIAL SCIENCE

CT3
SOURAV SIR'S CLASSES
100 MARKS 3HOURS
ATTEMPT ALL
PASS MARKS 55

1 A contestant on a game show is asked two questions. The probability that she gets the first question correct is 0.3 and the probability that she gets the second question correct is 0.4. Given that the probability that she gets both questions correct is 0.1, calculate the probability that:

(i) she gets either the first, the second or both questions right 4

(ii) she gets both questions wrong.

2 Customers at a restaurant may order any combination of chips, salad or onion rings. The probability that a customers chooses onion rings is 0.3, salad 0.4, chips and salad 0.l5, chips and onion rings 0.15, salad or onion rings 0.55, all three 0.05, none 0.2.
Calculate the probability a customer chooses:

(i) chips (ii) chips only 4

3 Two dice are thrown. Find the probability of rolling a 5 on both dice. 3

4 The punctuality of trains has been investigated by considering a number of train journeys. In the sample, 60% of trains had a destination of Manchester, 20% Birmingham and 20% Edinburgh. The probabilities of a train arriving late in Manchester, Edinburgh or Birmingham are 30% , 20% and 25% respectively.
If a late train is picked at random from the group under consideration, what is the probability that it terminated in Manchester? 4

5

The continuous random variable W has the PDF $f_W(w) = 12w^2(1-w)$, for $0 < w < 1$.
Calculate $P(W < \frac{1}{2})$ and determine an expression for $F_W(w)$.

3

6

A random variable X has PDF:

$$f_X(x) = \frac{1}{9}x^2 \qquad 0 < x < 3$$

6 Calculate $E(2X+1)$. 3

7

What is the probability that at least 9 out of a group of 10 people who have been infected by a serious disease will survive, if the survival probability for the disease is 70%?

3MARKS

8 The probability of having a male or female child is equal. A woman has two boys and a girl. What is the probability that her next two children are girls?
3

9 Among the 58 people applying for a job, only 30 have a particular qualification. If 5 of the group are randomly selected for a survey about the job application procedure, what is the probability that none of the group selected have the qualification?
Calculate the answer:
(i) exactly
(ii) using the binomial approximation 4

10

If the moment generating function of X is $M_X(t)$, then derive an expression for the moment generating function of $2X+3$. [2]

Hence, if X is normally distributed with mean μ and variance σ^2, derive the distribution of $2X+3$. [2]

A continuous random variable Y has PDF:

$$f(y) = \begin{cases} y(y-1)(y-2)+0.4 & 0 \le y \le 2 \\ c & 2 < y \le 4 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant.

Determine:

(i) the value of c

(ii) $E[Y]$

(iii) the standard deviation of Y.

11 2

12

An insurance company issues policies covering the loss of legs on pet spiders. Policies are issued to the owners and cover any number of spiders. The number of claims per policy in a year has a Poisson distribution with mean 0.6.

(i) Calculate the probability that a particular policyholder makes at least 3 claims this year. [2]

(ii) The policyholder in part (i) is known to have already made at least 1 claim this year. Calculate a revised probability that this policyholder makes at least 3 claims in total over this year. [2]

(iii) Calculate the probability that a portfolio of 5 independent policies gives rise to at least 3 claims this year. [2]

(iv) Obtain the probability that there is a wait of at least 4 months between claims on a single randomly selected policy. State any assumptions that you make. [3]

13

The probability generating function for a discrete random variable X is given by:

$$G_X(t) = 0.1(1+t)(4+t^2)$$

Calculate:

(i) $E(X)$ [1]

(ii) the standard deviation X [2]

(iii) $P(X \geq 2)$. [2]

14

An office manager wants to analyse the variability in the time taken for her typists to complete a given task. She has given seven typists the task and the results are as follows (in minutes):

15, 17.2, 13.7, 11.2, 18, 15.1, 14

The manager wants a 95% confidence interval for the true standard deviation of time taken of the form $\sigma > k$. Calculate the value of k. [4]

15

(i) In an opinion poll, a random sample is to be asked whether they favour closer ties with Europe. Determine the minimum sample size required to ensure that 95% confidence limits for the underlying population proportion are of the form "$\pm 5\%$", justifying any approximations used. [4]

(ii) 1,000 people took the opinion poll in part (i). 30% said "Yes" to closer ties with Europe, 50% said "No" and 20% said "Don't know". Calculate 95% confidence intervals for the proportion of the whole population holding each opinion. [4]

(iii) After an extensive advertising campaign by the government another opinion poll of 800 people was taken. Of those questioned, 35% said "Yes".

(a) Obtain a 90% confidence interval for the difference in proportions favouring closer ties with Europe before and after the campaign.

(b) Comment on your answer. [4]

[Total 12]

16

A general insurance company is debating introducing a new screening programme to reduce the claim amounts that it needs to pay out. The programme consists of a much more detailed application form that takes longer for the new client department to process. The screening is applied to a test group of clients as a trial whilst other clients continue to fill in the old application form. It can be assumed that claim payments follow a normal distribution.

The claim payments data for samples of the two groups of clients are (in £100 per year):

Without screening	24.5	21.7	35.2	15.9	23.7	34.2	29.3	21.1	23.5	28.3
With screening	22.4	21.2	36.3	15.7	21.5	7.3	12.8	21.2	23.9	18.4

(i) (a) Find a 95% confidence interval for the difference between the mean claim amounts.

(b) Comment on your answer. [6]

(ii) (a) Find a 95% confidence interval for the ratio of the population variances.

(b) Hence, comment on the assumption of equal variances required in part (i). [4]

(iii) Assume that the sample sizes taken from the clients with and without screening are always equal to keep processing easy. Calculate the minimum sample size so that the width of a 95% confidence interval for the difference between mean claim amounts is less than 10, assuming that the samples have the same variances as in part (i). [3]

17

The Chevalier de Méré, a seventeenth century gambler, thought that it paid to bet evens on the event:

A : you will get one or more sixes when four unbiased dice are thrown

In other words, the Chevalier thought that $P(A) > ½$.

(i) (a) Show that the probability of this event occurring is, in fact, 0.5177.

(b) The experiment of throwing four unbiased dice is performed 10 times and results in the event A occurring 8 times.

(1) Write down an equation which must be satisfied by p_L , a lower 95% confidence limit for $p = P(A)$.

(2) Verify that $p_L = 0.493$ satisfies this equation.

(3) Comment on this value of p_L relative to the true value of p as specified in part (i)(a). [6]

(ii) The experiment of throwing four unbiased dice is performed 1,000 times and results in the event A occurring Y times.

(a) Write down a large sample expression for p_L , a lower 95% confidence limit for $p = P(A)$.

(b) Determine how large Y would have to be for p_L to be greater than ½ .

(c) Using the true value of p as specified in part (i), calculate the probability that p_L will, in fact, be greater than ½ . [8]

(iii) For the situation where the experiment is performed 10,000 times:

(a) repeat part (ii)(b)

(b) repeat part (ii)(c)

and comment briefly on any difference between your two answers. [6]
[Total 20]