

1. (a) Evaluate the value of

$$\lim_{x \rightarrow \infty} \left(\frac{3x-4}{3x+2} \right)^{\frac{x+1}{3}}$$

- (b) Two squares are chosen at random on a chessboard. What is the probability that they have a side in common?

$$[6 + 9 = 15]$$

2. (a) A pizza chain offers cheese pizzas with a choice of these toppings: pepperoni, onion, sausage, mushrooms, and anchovies. Explain why the pizza chain can claim to offer 32 different pizzas.

- (b) Evaluate $\int \frac{dx}{\sin x + \sin 2x}$.

$$[5 + 10 = 15]$$

3. (a) The number of people in a small country town increases by 2% per year. If the population at the start of 1982 was 12,500 what is the predicted population at the start of the year 2022?

- (b) If two distinct integers a and b are picked from $\{1, 2, 3, 4 \dots 100\}$ and multiplied, what is the probability that the resulting number has EXACTLY 3 factors?

$$[5+10 = 15]$$

4. (a) The integers x, y, z are in a geometric sequence such that $y - x$ is a perfect cube. If $\log_{36} x^2 + \log_6 \sqrt{y} + 3 \log_{216} y^{\frac{1}{2}} z = 6$, find the value of $x + y + z$.

- (b) If $a + b + c = 0$ prove that $a^3 + b^3 + c^3 = 3abc$.

$$[9+6 = 15]$$

5. (a) The carbonitrided bushes need to meet the specifications on 3 output characteristics namely surface finish, case depth and dimensional variation. Otherwise, the bush is considered as a defective bush. Suppose a batch of 240 bushes have been carbonitrided and 30 of them not met the specification on surface finish, 15 on case depth, 9 on dimensional variation, 6 on both surface finish & case depth, 4 on surface finish & dimensional variation, 4 on case depth & dimensional variation and there is no bush not meeting specification on all the three characteristics. What is the chance that a bush randomly selected from the batch will be a defective bush?
- (b) Consider that the integers x_1, x_2, x_3, \dots are in Geometric Progression (G.P). The fourth term is 24. The ratio of third and fifth term is $\frac{1}{4}$. What is the second term?

[8+7 = 15]

6. (a) Consider the polynomial $g(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c and d are some real numbers. Suppose $x = 0$ is the only real solution of $g'(x) = 0$ and also that $g(-2) < g(2)$. Find the maximum of $g(x)$ in the interval $[-2, 2]$.
- (b) In a transaction process consisting of 3 activities namely allocation of the task, task processing and quality check. Nine employees, including five males and four females are working. Two of them are process leads who do allocation of the task, 4 executives perform the processing of the task and the remaining 3 are quality auditors who are engaged in quality check. Out of the 4 female employees, one is a process lead, two are processing executives and the remaining one is a quality auditor. Suppose the chance of allocating a task is equal among the process leads, the chance of processing is equal among 4 process executives and that of quality check is equal among quality auditors, what is the chance that a task is completely handled (allocation, processing & quality check) by female employees?

[8+7 = 15]

7. (a) ‘Success’ in a roll of a six-faced balanced die is defined as hitting a six (or the die comes up showing six dots). Let p denote the probability of ‘Success’. If the probability of obtaining at least one success in n repeated rolls of this die is greater than or equal to 0.9, then find the smallest possible n .
- (b) In an agile software development process, the sprint productivity (SP) is related to developer experience (DE), review time (RT) and code reuse (CR) as follows:

$$SP = 1.33 + 0.05DE - 0.01RT + 0.02CR$$

The mean and standard deviation of developer experience, review time and code reuse is given in the table below. What will be the variance of the sprint productivity?

Variable	Mean	Standard Deviation
Developer Experience	4	0.5
Review Time	25	2
Code Reuse	40	10

[10+5 = 15]

8. (a) Suppose the solutions of the equation $x^2 + ax + 2 = 0$ (where a is a real number) are real numbers and the difference between the solutions is less than $\sqrt{8}$. Find the set of possible values of a satisfying this condition.
- (b) According to the service level agreement between the client and an IT company, the daily average resolution time of tickets should be within 24 hours and if any ticket takes more than 30 hours, the IT company has to pay a penalty of 5 dollars to the client. On a particular day, the IT company received 6 tickets to resolve and later it was found that the average resolution time for that day was 24 hours. Further investigation revealed that 2 tickets out of 6 has taken more than 24 hours to resolve. When the resolution times of those two tickets are removed, the resolution time average reduced to 23 hours. Whether the IT company paid any penalty on that day? If yes, how much?

[8+7 = 15]