SOURAV SIR'S CLASSES 9836793076

ACTUARIAL SCIENCE

PROBABILITY & MATHEMATICAL STATISTICS

EXAM 3

100 MARKS 3HOURS

1.

A coin is selected at random from a pair of coins and tossed. Coin 1 is a double-headed coin (*ie* a head on both sides). Coin 2 is a standard unbiased coin.

The result of the toss is a head. What is the probability that it was coin 1 which was tossed? 3

2

For a certain type of insurance business, the number of claims per policy in a year has a Poisson distribution with mean 0.4

Consider a policy, which you know, has given rise to at least one claim in the last year. The probability that this policy has in fact given rise to exactly two claims in the least year is:

A 0.054 B 0.163 C 0.330 D 0.992 2

3

If a random variable X has a mean of 3 and standard deviation of 2, calculate:

(i) $E[2X-4]$

(ii) $\text{var}[3X+2]$

(iii) $\text{var}[3-4X]$

(iv) $E\left[\dfrac{3X+2}{4}\right]$

4

4

Use a series expansion to find $E(X), E(X^2)$ and $E(X^3)$ of a random variable, X, with MGF given by:

$$M_X(t) = \left(1 - \tfrac{t}{5}\right)^{-1} \qquad t < 5$$

4

5

The continuous random variables U and V have the joint probability density function:

$$f_{U,V}(u,v) = \frac{2u+v}{3000}, \text{ where } 10 < u < 20 \text{ and } -5 < v < 5$$

Find $P(10 < U < 15, V > 0)$. 5

6

The number of claims arising in a month under a home insurance policy follows a Poisson distribution with mean 0.075. Calculate the approximate probability that at least 50 claims in total arise in a month under a group of 500 independent such policies.

4

7

An investigation was carried out into the number of hours lost due to stress-related illnesses at various companies. The results for a random sample of individuals from five companies are shown below.

	company				
	A	*B*	*C*	*D*	*E*
	30	42	65	67	70
	25	57	46	58	63
	12	47	55	81	80
	23	30	27		
	16				
totals	106	176	193	206	213

$$\sum\sum y_{ij}^2 = 50{,}354$$

Use analysis of variance to test for differences between companies. You should present your results in the form of an ANOVA table. 8

8

A random sample, $x_1, \ldots, x_{10}$, from a normal population gives the following values:

9.5 18.2 4.69 3.76 14.2 17.13 15.69 13.9 15.7 7.42

$$\sum x_i = 120.19 \qquad \sum x_i^2 = 1{,}693.6331$$

(i) Test at the 5% level whether the mean of the whole population is 15 if the variance is:

(a) unknown

(b) 20. [5]

(ii) Test at the 5% level whether the population variance is 20. [3]
[Total 8]

9

An analysis using the simple linear regression model based on 19 data points gave:

$s_{xx} = 12.2 \qquad s_{yy} = 10.6 \qquad s_{xy} = 8.1$

(i) (a) Calculate $\hat{\beta}$.

(b) Test whether β is significantly different from zero. [4]

(ii) (a) Calculate r.

(b) Test whether ρ is significantly different from zero. [4]

(iii) Comment on the results to your tests from part (i) and (ii). [2]
[Total 10]

10

A general insurance company is debating introducing a new screening programme to reduce the claim amounts that it needs to pay out. The programme consists of a much more detailed application form that takes longer for the new client department to process. The screening is applied to a test group of clients as a trial whilst other clients continue to fill in the old application form. It can be assumed that claim payments follow a normal distribution.

The claim payments data for samples of the two groups of clients are (in £100 per year):

Without screening	24.5	21.7	45.2	15.9	23.7	34.2	29.3	21.1	23.5	28.3
With screening	22.4	21.2	36.3	15.7	21.5	7.3	12.8	21.2	23.9	18.4

(i) Test the hypothesis that the new screening programme reduces the mean claim amount. [5]

(ii) Formally test the assumption of equal variances required in part (i). [3]
[Total 8]

11

An insurer believes that the distribution of the number of claims on a particular type of policy is binomial with parameters $n = 3$ and p. A random sample of the number of claims on 153 policies revealed the following results:

Number of claims	0	1	2	3
Number of policies	60	75	16	2

(i) Derive the maximum likelihood estimate of p. [4]

(ii) Carry out a goodness of fit test for the binomial model specified in part (i) for the number of claims on each policy. [5]

[Total 9]

12

Insurance claims (in £) arriving at an office over the last month have been analysed. The results are as follows:

Claim size, c	$0 \le c < 500$	$500 \le c < 1000$	$1000 \le c < 2500$	over 2,500
No. of claims	75	51	22	5

(i) Assuming that the maximum claim amount is £10,000:

(a) calculate the sample mean of the data

(b) test at the 5% level whether an exponential distribution with parameter λ is an appropriate distribution for the claim sizes. You should estimate the value of λ using the method of moments. [6]

(ii) An actuary decides to investigate whether claim sizes vary according to the postcode of residence of the claimant. She splits the data into the three different postcodes observed. The results for the first two postcodes are given below:

Postcode 1:

Claim size, c	$0 \le c < 500$	$500 \le c < 1000$	$1000 \le c < 2500$	over 2,500
No. of claims	23	14	7	3

Postcode 2:

Claim size, c	$0 \le c < 500$	$500 \le c < 1000$	$1000 \le c < 2500$	over 2,500
No. of claims	30	16	11	1

Test at the 5% level whether claim sizes are independent of the postcodes. [8]

[Total 14]

13

The table below shows the approximate monetary equivalent of the annual pay and benefit packages (in £000s) of two independent samples of actuarial students, one for male students and one for female students, all of whom have passed 2 exams.

Male students	21	18	24	25	21	32	23	21
Female students	22	22	20	19	24	28	27	22

Draw boxplot diagrams for the pay packages of the male and female students. Use these to compare and contrast the two distributions. [6]

14

(i) The records of cash payments received in a week by two clerks are summarised below:

Clerk	No. of payments	Sample mean	Sample standard deviation
A	1,000	£250	£30
B	1,500	£240	£25

Calculate the overall mean and standard deviation of the 2,500 payments. [4]

(ii) The marks (%) of a sample of 20 students from a large class in a recent examination had a sample mean of 43 and a sample standard deviation of 6. The marks were subsequently adjusted – each mark was multiplied by 1.3 and the result was then increased by 10. Calculate the sample standard deviation of the adjusted marks. [2]

[Total 6]

15

A general insurance portfolio contains 100 policies. The probability that any policy in the portfolio makes one or more claims is 0.2, independent of any other policy.

The number of claims arising on a policy is modelled as a Type 1 negative binomial with $k = 1$ and $p = 0.6$, independently for each policy and independent of the number of policies giving rise to claims.

Calculate the mean and standard deviation of the total number of claims for this portfolio. [4]

16

The time to complete a tricky pension review is normally distributed with mean 8 hours and standard deviation 2 hours. Calculate the probability that the times taken for two randomly selected tricky reviews differ by no more than 3 hours. [4]