2022

Booklet number **00181** TEST CODE: **QMA** ~~PMA~~

Forenoon **Time: 2 hours**

- This test contains thirty (30) multiple-choice questions (MCQs).
- The questions are to be answered in a separate *Optical Mark Recognition* (OMR) Answer Sheet.
- Please write your *Name*, *Registration Number*, *Test Centre*, *Test Code* and the *Number of this Question Booklet* in the appropriate places on the OMR Answer Sheet. Please do not forget to put your signature in the designated place.
- For each of the questions there are four suggested answers, of which only one is correct. For each question, indicate your choice of the correct answer by darkening the appropriate circle (●) completely on the OMR Answer Sheet, using ball-point pen with BLACK ink only.
- You will score
  - 4 marks for each correctly answered question,
  - 0 mark for each incorrectly answered question, and
  - 1 mark for each un-attempted question.
- ALL ROUGH WORK MUST BE DONE ONLY IN THE SPACE AVAILABLE ON THIS QUESTION BOOKLET.
- USE OF CALCULATORS, MOBILE PHONES AND ALL TYPES OF ELECTRONIC DEVICES IS STRICTLY PROHIBITED.

STOP! WAIT FOR THE SIGNAL TO START.

1. The formula for the time a traffic light remains yellow is $t = \frac{1}{8}s + 1$ where t is the time in seconds and s is the speed limit in miles/hour (mph). What is the speed limit if the traffic light remains yellow for 4 seconds?

(A) 24 mph (B) 31 mph (C) 32 mph (D) 40 mph

2. Let a, b, c, d and e be integers such that $a = 6b = 12c$ and $2b = 9d = 12e$, then which of the following pairs contains a number that is not an integer ?

(A) $\left(\frac{a}{27}, \frac{b}{e}\right)$ (B) $\left(\frac{a}{36}, \frac{c}{e}\right)$

(C) $\left(\frac{a}{12}, \frac{bd}{18}\right)$ (D) $\left(\frac{a}{6}, \frac{c}{d}\right)$

3. Find the value of z from the following equations

$$(x + y)(x + y + z) = 21$$
$$(y + z)(x + y + z) = 42$$
$$(z + x)(x + y + z) = 35$$

(A) 4 (B) -4 (C) ± 4 (D) None of these

4. A two-wheeler manufacturer has to select a subcontractor for the aluminium plating of the wheel rims. Five aluminium plating subcontractors have been short-listed and given 50 wheel rims to each subcontractor for aluminium plating. After aluminium plating, the wheel rims are thoroughly checked and the ones with plating not satisfactory are rejected. The number of wheel rims rejected due to defective plating is less than 10 for each subcontractor. If the median rejection is 5 and mode is 4, what is the minimum rejection?

(A) 0 (B) 2 (C) 3 (D) None of these

5. The number of four-digit numbers that can be formed using the digits 0, 2, 3, 4, 5, 6, 7 that are divisible by 5 and repetition is not allowed are

(A) 240 (B) 230 (C) 220 (D) 200

6. If p is a real number and if the middle term in the expansion of $\left(\frac{p}{2}+2\right)^8$ is 1120, then the value of p is

(A) ± 3 (B) ± 2 (C) ± 1 (D) None of these

7. The value of

$$\lim_{(x,y)\to(0,0)} xy\frac{x^2-y^2}{x^2+y^2}$$

is

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) 2

8. The length of six line segments are 2, 3, 4, 5, 6, & 7 units. The number of triangles that can be formed by these lines is

(A) ${}^6C_3 - 7$ (B) ${}^6C_3 - 6$ (C) ${}^6C_3 - 5$ (D) ${}^6C_3 - 8$

9. Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to

(A) 25 (B) 34 (C) 42 (D) 41

10. **Given** $(y) = \frac{y+4}{2y-5}$, find $z^{-1}(y)$

(A) $\frac{4+5y}{2y-1}$ (B) $\frac{5+4y}{2y-1}$ (C) $\frac{2+3y}{5y-3}$ (D) $\frac{2+4y}{3y-6}$

11. A balloon takes off from a location that is 24 feet above sea level. It rises 45 feet/minute. Choose the correct equation to model the balloon's elevation h as a function of time t

(A) $h = 24t + 45$ (B) $t = 24h + 45$

(C) $h = 45t + 24$ (D) $t = 45h + 24$

12. If the product of two matrices

$$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

and

$$B = \begin{bmatrix} \cos^2\varphi & \cos\varphi\sin\varphi \\ \cos\varphi\sin\varphi & \sin^2\varphi \end{bmatrix}$$

is a null matrix, then θ and φ differ by

(A) Odd multiple of π (B) Even multiple of π

(C) Odd multiple of $\frac{\pi}{2}$ (D) Even multiple of $\frac{\pi}{2}$

13. If $f(x) = \frac{9^x}{9^x+3}$ then the sum of the terms

$$f\left(\frac{1}{1996}\right) + f\left(\frac{2}{1996}\right) + \cdots + f\left(\frac{1995}{1996}\right)$$

is

(A) $996\frac{1}{2}$ (B) $997\frac{1}{2}$ (C) $998\frac{1}{2}$ (D) $999\frac{1}{2}$

14. If $a_1, a_2, \ldots, a_{21}$ are in arithmetic progression and $\sum_{i=1}^{21} a_i = 693$, then $\sum_{i=0}^{10} a_{2i+1}$ is

(A) 361 (B) 396 (C) 363 (D) 292

15. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

(A) 1 square unit (B) 2 square units

(C) $2\sqrt{2}$ square units (D) 4 square units

16. A day's production of 850 manufactured parts contains 50 parts that do not meet customer requirements. Two parts are selected randomly without replacement from the batch. The probability that the second part is defective given that the first part is defective is

(A) 0.0032 (B) $\frac{49}{849}$ (C) $\frac{50}{850} \times \frac{49}{850}$ (D) $\frac{50}{849}$

17. If $ab = 2a + 3b, a > 0, b > 0$, then the minimum value of ab is

(A) 18 (B) 36 (C) 24 (D) $\frac{1}{4}$

18. A seven-digit number is formed using the digit 3, 3, 4, 4, 4, 5, 5 The probability, that number so formed is divisible by 2, is

(A) $\frac{6}{7}$ (B) $\frac{4}{7}$ (C) $\frac{3}{7}$ (D) $\frac{1}{7}$

19. Solve: $y - ye^{5y+2} = 0$

(A) 1 and $-\frac{1}{5}$ (B) 0 and $-\frac{5}{2}$

(C) 0 and $-\frac{2}{5}$ (D) 1 and $-\frac{5}{2}$

20. Find the opposite and reciprocal of the number 0.38

(A) $\frac{50}{19}$, $\frac{19}{50}$ (B) -0.38, $\frac{50}{19}$

(C) -0.38, $\frac{19}{50}$ (D) $\frac{50}{19}$, 0.38

21 The sum of eigenvalues of the matrix $\begin{bmatrix} 3 & 4 \\ x & 1 \end{bmatrix}$ for real and negative values of x is

(A) greater than 0 (B) less than 0

(C) 0 (D) dependent on the value of x

22 From a group of 3 members of party A, 2 members of party B, and 1 member of party C, a committee of two people is to be selected randomly. What is the probability that the committee has 1 member of party A and a member of party B?

(A) 0.3 (B) 0.4 (C) 0.5 (D) 0.6

23. An experiment succeeds twice as often it fails. Then, the probability that in the next four trials there will be at least 2 successes is

(A) $\frac{1}{9}$ (B) $\frac{8}{9}$ (C) $\frac{5}{9}$ (D) $\frac{2}{9}$

24. The value of

$$\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$$

is

(A) $\frac{e}{2}$ (B) $\frac{1}{2}$ (C) $-\frac{e}{2}$ (D) $\frac{3}{2}$

25. Three distinct numbers are selected from the first 100 natural numbers. The probability that all the three numbers are divisible by 2 and 3 is

(A) $\frac{4}{25}$ (B) $\frac{4}{35}$ (C) $\frac{4}{55}$ (D) $\frac{4}{1155}$

26. Given $x = cy + bz, y = az + cx, z = bx + ay$ where x, y, z are not all zero. The value of $a^2 + b^2 + c^2 + 2abc$ is

(A) 1 (B) 0 (C) 2 (D) 3

27. The value of

$$\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$$

is

(A) 12 (B) -18 (C) -10 (D) -12

28. If $f(\pi) = 2$ and $\int_0^{\pi} (f(x) + f''(x)) \sin x \, dx = 5$ then $f(0)$ is equal to (assume $f(x)$ is continuous in $[0, \pi]$)

(A) 0 (B) 1 (C) 2 (D) 3

29. If

$$P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$$

is the adjoint of a 3×3 matrix A and $|A| = 4$, then the value of α is

(A) 4 (B) 11 (C) 5 (D) 0

30. The coefficient of x^7 in the expression

$(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \cdots + x^{10}$

is

(A) 420 (B) 330 (C) 210 (D) 120

31

1. (a) Twelve points are located on the circumference of a circle. Lines are drawn to connect all possible pairs of points. How many lines are drawn?

(b) Find the length of the arc of the parabola $x^2 = 4y$ from the vertex to the point $x = 2$, where $x \in R$ and $y \in R$.

[7 + 8 = 15]

2. (a) A company manufacturing industrial enclosures is interested in optimizing the powder coating process to minimize the variation in coating thickness around the target value of 56 microns. The models developed for thickness variance and the expected value of thickness are given below. Identify the optimum values of conductivity and powder output which would result in an expected thickness of 56 microns with a minimum variance? To execute the powder coating process, the conductivity and powder output needs to be greater than zero.

$$\text{Variance} = 9.5 + 2 \times \text{Conductivity}^2 + 3 \times (\text{Powder output})^2$$

$$\text{Expected Thickness} = 4 \times \text{Conductivity} + 12 \times (\text{Powder output})$$

(b) What is the interval on which the function $f(x) = \frac{\log(x)}{x}$, $x \in (0, \infty)$ is increasing?

[8 + 7 = 15]

3. (a) A student is allowed to select at most n books from a collection of $(2n + 1)$ books. If the number of ways in which he can select at least one book is 63, find the value of n.

(b) Show that the sequence $\{S_n\}$, where $S_n = \left(1 + \frac{1}{n}\right)^2$, is convergent and that $\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$ lies between 2 and 3.

[7 + 8 = 15]

4. (a) If the third, fourth and fifth terms in the expansion of $(a + x)^n$ be 84, 280 and 560 respectively, find the value of a, x and n.

(b) If $x_1, x_2, \ldots x_{20}$ are in harmonic progression and $x_1, 2, x_{20}$ are in geometric progression, find the value of $\sum_{r=1}^{19} x_r x_{r+1}$.

[7 + 8 = 15]

5. (a) John wishes to get from town A to town C via town B. There are three roads connecting town A to town B and 4 roads connecting town B to town C. In how many ways can John get from town A to town C?

(b) Find the values of λ and μ for which the system of equation

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have no solution.

[6 + 9 = 15]

6. (a) A 1-inch-diameter coin is thrown on a table covered with a grid of lines two inches apart. What is the probability that the coin lands in a square without touching any of the lines of the grid?

(b) One of the subjects of a postgraduate course offered by a reputed institute is handled online by a faculty from another city. Suppose the video conferencing software used by the faculty is 99% reliable, his internet connection is 98% reliable and the internet connectivity at the institute is also 98% reliable, what is the chance of having failure-free online classes? The online classes will work only if the video conferencing software and internet connectivity at the faculty premise and the institute works. Suppose another video conferencing software is available in the market which is also 99% reliable and an alternative internet service provider is

nbers that can be formed using n
divisible by 5 and

available at faculty's city with 98% reliability. Can purchasing and keeping one more video conferencing software or internet facility or both and keeping them as standby improve the chance of failure-free operation of the online class to 98%?

[7 + 8 =15]

7. (a) A and B throw a die alternatively till one of them throws a '6' and wins the game. Find their respective probabilities of winning, if A starts the game.

(b) Calculate the mean deviation for the arithmetic progression $a, a+d, a+2d, a+3d, \ldots, a+2nd$.

[7 + 8 = 15]

8. (a) Customers are used to evaluating preliminary product designs. In the past 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

i. What is the probability that a product attains a good review?

ii. If a new design attains a good review, what is the probability that it will be a highly successful product?

iii. If a product does not attain a good review, what is the probability that it will be a highly successful product?

(b) If

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

find m so that $A^2 = mA - 2B$.

[8 + 7 = 15]