- There are 6 questions: 2 in Group A, 2 in Group B, and 2 in Group C.
- Answer **4** questions in total and **at least 1** question from each Group.
- Each question carries equal marks.
- The maximum possible score is **100**.

Group A

1. Suppose a government agency has a monopoly in the provision of internet connections. The marginal cost of providing internet connections is $\frac{1}{2}$, whereas the inverse demand function is given by: $p = 1 - q$. The official charge per connection is set at 0; thus, the state provides a subsidy of $\frac{1}{2}$ per connection. However, the state can only provide budgetary support for the supply of 0.4 units, which it raises through taxes on consumers. Bureaucrats in charge of sanctioning internet connections are in a position to ask for bribes, and consumers are willing to pay them in order to get connections. Bureaucrats cannot, however, increase supply beyond 0.4 units.

   (a) Find the equilibrium bribe rate per connection and the social surplus.

   (b) Now suppose the government agency is privatized and the market is deregulated; however, due large fixed costs of entry relative to demand, the privatized company continues to maintain its monopoly. Find the new equilibrium price, bribe rate and social surplus, specifying whether privatization increases or reduces them.

   (c) Suppose now a technological innovation becomes available to the privatized monopoly, which reduces its marginal cost of providing an internet connection to c, $0 < c < \frac{1}{2}$. Find the range of values of c for which privatization increases consumers' surplus.

2. Consider an exchange economy consisting of two individuals 1 and 2, and two goods, X and Y. The utility function of individual 1 is $U_1 = X_1 + Y_1$, and that of individual 2 is $\min\{X_2, Y_2\}$, where X_i (resp. Y_i) is the amount of X (resp. Y) consumed by individual i, where $i = 1, 2$. Individual 1 has 4 units of X and 8 units of Y, and individual 2 has 6 units of X and 4 units of Y to begin with.

   (a) What is the set of Pareto optimal outcomes in this economy? Justify your answer.

   (b) What is the competitive equilibrium in this economy? Justify your answer.

(c) Are the perfectly competitive equilibria Pareto optimal?

(d) Now consider another economy where everything is as before, apart from individual 2's preferences, which are as follows: (a) among any two bundles consisting of X and Y, individual 2 prefers the bundle which has a larger amount of commodity X irrespective of the amount of commodity Y in the two bundles, and (b) between any two bundles with the same amount of X, she prefers the one with a larger amount of Y. Find the set of Pareto optimal outcomes in this economy.

Group B

1. An economy comprises of a consolidated household sector, a firm sector and the government. The household supplies labour (L) to the firm. The firm produces a single good (Y) by means of a production function $Y = F(L)$, $F'(L) > 0$, $F''(L) < 0$, and maximizes profits $\Pi = PY - WL$, where P is the price of Y and W is the wage rate. The household, besides earning wages, is also entitled to the profits of the firm. The household maximizes utility (U), given by

$$U = \tfrac{1}{2}\ln C + \tfrac{1}{2}\ln\left(\tfrac{M}{P}\right) - d(L),$$

where C is consumption of the good and $\frac{M}{P}$ is real balance holding. The term $d(L)$ denotes the disutility from supplying labour with $d'(L) > 0$, $d''(L) > 0$. The household's budget constraint is given by:

$$PC + M = WL + \Pi + \overline{M} - PT,$$

where $\overline{M}$ is the money holding the household begins with, M is the holding they end up with and T is the real taxes levied by the government. The government's demand for the good is given by G. The government's budget constraint is given by:

$$M - \overline{M} = PG - PT.$$

Goods market clearing implies $Y = C + G$.

(a) Prove that $\frac{dY}{dG} \in (0,1)$, and that government expenditure crowds out private consumption (i.e., $\frac{dC}{dG} < 0$).

(b) Show that everything else remaining the same, a rise in $\overline{M}$ leads to an equiproportionate rise in P.

2. Consider an IS-LM model where the sectoral demand functions are given by

$$\begin{aligned} C &= 90 + 0.75Y, \\ G &= 30, \\ I &= 300 - 50r, \\ \left(\frac{M}{P}\right)_d &= 0.25Y - 62.5r, \\ \left(\frac{M}{P}\right)_s &= 500. \end{aligned}$$

Any disequilibrium in the international money market is corrected instantaneously through a change in r. However, any disequilibrium in the goods market, which is corrected through a change in Y, takes much longer to be eliminated.

(a) Consider an initial situation where $Y = 2500$, $r = \frac{1}{5}$. What is the change in the level of I that must occur before there is any change in the level of Y ?

(b) Draw a graph to explain your answer.

(c) Calculate the value of (r, Y) that puts both the money and goods market in equilibrium. What is the value of investment at this point compared to $(r = 0.2, Y = 2500)$?

Group C

1. Answer the following questions.

(a) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as

$$f(x) = \frac{|x|}{2x} \ \forall\, x \in \mathbb{R} \setminus \{0\}.$$

Can $f(0)$ be defined in a way such that f is continuous at 0? Justify your answer.

(b) Consider the following optimization problem:

$$\max_{x \in [0,\beta]} x(1 - x),$$

where $\beta \in [0, 1]$. Let x^* be an optimal solution of the above optimization problem. For what values of β will we have $x^* = \beta$?

(c) A firm is producing two products a and b. The market price (per unit) of a and b are respectively 3 and 2. The firm has resources to produce only 10 units of a and b together. Also, the quantity of a produced cannot exceed double the quantity of b produced. What is the revenue-maximizing production plan (i.e., how many units of a and b) of the firm?

2. Answer the following questions.

   (a) A slip of paper is given to person A, who marks it with either $(+)$ or $(-)$. The probability of her writing $(+)$ is $\frac{1}{3}$. Then, the slip is passed sequentially to B, C, and D. Each of them either changes the sign on the slip with probability $\frac{2}{3}$ or leaves it as it is with probability $\frac{1}{3}$.

      i. Compute the probability that the final sign is $(+)$ if A wrote $(+)$.

      ii. Compute the probability that the final sign is $(+)$ if A wrote $(-)$.

      iii. Compute the probability that A wrote $(+)$ if the final sign is $(+)$.

   (b) There are n houses on a street numbered $h_1, \ldots, h_n$. Each house can either be painted BLUE or RED.

      i. How many ways can the houses $h_1, \ldots, h_n$ be painted?

      ii. Suppose $n \geq 4$ and the houses are situated on n points on a circle. There is an additional constraint on painting the houses: exactly two houses need to be painted BLUE and they cannot be next to each other. How many ways can the houses $h_1, \ldots, h_n$ be painted under this new constraint?

      iii. How will your answer to the previous question change if the houses are located on n points on a line.