ACTUARIAL SCIENCE

CT3 EXAMINATION PAPER

TEST 02

100 MARKS 3HOURS

BY

SOURAV SIR'S CLASSES

1..

A random variable X has a $Poi(3.6)$ distribution.

(i) Calculate the mode of the probability distribution. [2]

(ii) Calculate the standard deviation of the distribution. [1]

(iii) State, with reasons, whether the distribution is positively or negatively skewed. [1]
[Total 4]

2

(i) If the moment generating function of X is $M_X(t)$, then derive an expression for the moment generating function of $2X+3$. [2]

(ii) Hence, if X is normally distributed with mean μ and variance σ^2, derive the distribution of $2X+3$. [2]
[Total 4]

3

An actuarial recruitment company places adverts in three publications with probabilities of 0.2, 0.3 and 0.5 respectively.

The probability that the recruitment company gets an enquiry from an advert in the first publication is 0.001. The probabilities for the other two publications are 0.002 and 0.004 respectively.

Given that the company has just received an enquiry, calculate the probability that it came from an advert in the first publication. [3]

4

A continuous random variable Y has PDF:

$$f(y) = \begin{cases} y(y-1)(y-2)+0.4 & 0 \le y \le 2 \\ c & 2 < y \le 4 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant.

Determine:

(i) the value of c [3]

(ii) $E[Y]$ [2]

(iii) the standard deviation of Y. [2]
[Total 7]

5

X is normally distributed with mean μ and variance σ^2. Use generating functions to find the fourth central moment of X. [3]

6

Let X and Y have joint density function:

$$f_{X,Y}(x,y) = \frac{4}{5}\left(3x^2 + xy\right) \qquad 0 < x < 1, 0 < y < 1$$

Determine:

(i) the marginal density function X [2]

(ii) the conditional density function of Y given $X = x$ [1]

(iii) the covariance of X and Y. [5]
[Total 8]

7

The time taken to process simple home insurance claims has a mean of 20 mins and a standard deviation of 5 mins. Stating clearly any assumptions that you make, calculate the probability that the:

(i) sample mean of 5 claims is less than 15 mins [2]

(ii) sample mean of 50 claims is greater than 22 mins [2]

(iii) sample variance of 5 claims is greater than 6.65 mins [2]

(iv) sample standard deviation of 30 claims is less than 7 mins [2]

(v) both (i) and (iii) hold. [1]
[Total 9]

8

(i) $X_1, \ldots, X_n$ are independent and identical $Gamma(\alpha, \lambda)$ distribution. Show, using moment generating functions, that $\bar{X}$ has a $Gamma(n\alpha, n\lambda)$ distribution.
[3]

(ii) If the random variable T, representing the total lifetime of an individual light bulb, has an $Exp(\lambda)$ distribution, where $1/\lambda = 2{,}000$ hours, calculate the probability that the average lifetime of 10 bulbs will exceed 4,000 hours. [3]
[Total 6]

9

(i) Two discrete random variables, X and Y, have the following joint probability function:

		X			
		1	2	3	4
Y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2

Determine $\text{var}(X \mid Y = 2)$. [3]

(ii) Let U and V have joint density function:

$$f_{U,V}(u,v) = \tfrac{48}{67}\left(2uv - u^2\right) \qquad 0 < u < 1, \tfrac{u}{2} < v < 2$$

Determine $E(U \mid V = v)$. [3]
[Total 6]

10

Heights of males with classic congenital adrenal hyperplasia (CAH) are assumed to be normally distributed.

Determine the minimum sample size to ensure that a 95% confidence interval for the mean height has a maximum width of 10cm, if:

(i) a previous sample had a standard deviation of 8.4 cm [3]

(ii) the population standard deviation *is* 8.4 cm. [3]
[Total 6]

11

A discrete random variable has a probability function given by:

x	2	4	5
$P(X = x)$	$\frac{1}{8}+2\alpha$	$\frac{1}{2}-3\alpha$	$\frac{3}{8}+\alpha$

(i) Give the range of possible values for the unknown parameter α. [1]

A random sample of 30 observations gave respective frequencies of 7, 6 and 17.

(ii) Calculate the method of moments estimator of α. [3]

(iii) Write down an expression for the likelihood of these data and hence show that the maximum likelihood estimate $\hat{\alpha}$ satisfies the quadratic equation:

$$180\hat{\alpha}^2 + \frac{111}{8}\hat{\alpha} - \frac{91}{32} = 0$$ [5]

(iv) Hence determine the maximum likelihood estimate and explain why the second root is rejected as a possible estimate of α. [3]
[Total 12]

12

(i) In an opinion poll, a random sample is to be asked whether they favour closer ties with Europe. Determine the minimum sample size required to ensure that 95% confidence limits for the underlying population proportion are of the form " ±5% ", justifying any approximations used. [4]

(ii) 1,000 people took the opinion poll in part (i). 30% said "Yes" to closer ties with Europe, 50% said "No" and 20% said "Don't know". Calculate 95% confidence intervals for the proportion of the whole population holding each opinion. [4]

(iii) After an extensive advertising campaign by the government another opinion poll of 800 people was taken. Of those questioned, 35% said "Yes".

(a) Obtain a 90% confidence interval for the difference in proportions favouring closer ties with Europe before and after the campaign.

(b) Comment on your answer. [4]

[Total 12]

13

A general insurance company is debating introducing a new screening programme to reduce the claim amounts that it needs to pay out. The programme consists of a much more detailed application form that takes longer for the new client department to process. The screening is applied to a test group of clients as a trial whilst other clients continue to fill in the old application form. It can be assumed that claim payments follow a normal distribution.

The claim payments data for samples of the two groups of clients are (in £100 per year):

Without screening	24.5	21.7	45.2	15.9	23.7	34.2	29.3	21.1	23.5	28.3
With screening	22.4	21.2	36.3	15.7	21.5	7.3	12.8	21.2	23.9	18.4

(i) Test the hypothesis that the new screening programme reduces the mean claim amount. [5]

(ii) Formally test the assumption of equal variances required in part (i). [3]

[Total 8]

14

In an extrasensory perception experiment carried out in a live television interview, the interviewee who claimed to have extrasensory powers was required to identify the pattern on each of 10 cards, which had been randomly assigned with one of five different patterns. The cards were visible only to the audience who were asked to "transmit" the patterns to the interviewee. When the interviewee failed to identify any of the cards correctly, she claimed that this was clear proof of the existence of ESP, since there was a strong mind in the audience who was willing her to get the answers wrong.

(i) State the hypotheses implied by the interviewee's conclusion and carry out a 5% test on this basis. Comment on your answer. [3]

(ii) State precisely the hypotheses that the interviewer could have specified before the experiment to prevent the interviewee from "cheating" in this way, and determine the number of cards that would have to be identified correctly to demonstrate the existence of ESP at the 5% level. [2]

[Total 5]

15

The sizes of claims made to an insurance company have a lognormal distribution with mean £1,800 and standard deviation £280.

(i) Calculate the probability that a claim is more than £2,000. [4]

An actuary goes through a pile of claims noting those which are in excess of £2,000.

(ii) Calculate the probability that she examines eight claims before she finds three that are in excess of £2,000. [2]

[Total 6]