1. Let $0 < x < \frac{1}{6}$ be a real number. When a certain biased dice is rolled, a particular face F occurs with probability $\frac{1}{6} - x$ and and its opposite face occurs with probability $\frac{1}{6} + x$; the other four faces occur with probability $\frac{1}{6}$. Recall that opposite faces sum to 7 in any dice. Assume that the probability of obtaining the sum 7 when two such dice are rolled is $\frac{13}{96}$. Then, the value of x is:

   (A) $\frac{1}{8}$ (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{27}$.

2. An office has 8 officers including two who are twins. Two teams, Red and Blue, of 4 officers each are to be formed randomly. What is the probability that the twins would be together in the Red team?

   (A) $\frac{1}{6}$ (B) $\frac{3}{7}$ (C) $\frac{1}{4}$ (D) $\frac{3}{14}$

3. Suppose Roger has 4 identical green tennis balls and 5 identical red tennis balls. In how many ways can Roger arrange these 9 balls in a line so that no two green balls are next to each other and no three red balls are together?

   (A) 8 (B) 9 (C) 11 (D) 12

4. The number of permutations σ of $1, 2, 3, 4$ such that $|\sigma(i) - i| < 2$ for every $1 \leq i \leq 4$ is

   (A) 2 (B) 3 (C) 4 (D) 5.

5. Let $f(x)$ be a degree 4 polynomial with real coefficients. Let z be the number of real zeroes of f, and e be the number of local extrema (i.e., local maxima or minima) of f. Which of the following is a possible (z, e) pair?

   (A) $(4, 4)$ (B) $(3, 3)$ (C) $(2, 2)$ (D) $(0, 0)$

6. A number is called a palindrome if it reads the same backward or forward. For example, 112211 is a palindrome. How many 6-digit palindromes are divisible by 495?

   (A) 10 (B) 11 (C) 30 (D) 45

7. Let A be a square matrix of real numbers such that $A^4 = A$. Which of the following is true for every such A?

   (A) $\det(A) \neq -1$
   (B) A must be invertible.
   (C) A can not be invertible.
   (D) $A^2 + A + I = 0$ where I denotes the identity matrix.

8. Consider the real-valued function $h : \{0, 1, 2, \ldots, 100\} \to \mathbb{R}$ such that $h(0) = 5$, $h(100) = 20$ and satisfying $h(i) = \frac{1}{2}(h(i+1) + h(i-1))$, for every $i = 1, 2, \ldots, 99$. Then, the value of $h(1)$ is:

   (A) 5.15 (B) 5.5 (C) 6 (D) 6.15.

9. An up-right path is a sequence of points $\mathbf{a}_0 = (x_0, y_0), \mathbf{a}_1 = (x_1, y_1), \mathbf{a}_2 = (x_2, y_2), \ldots$ such that $\mathbf{a}_{i+1} - \mathbf{a}_i$ is either $(1, 0)$ or $(0, 1)$. The number of up-right paths from $(0, 0)$ to $(100, 100)$ which pass through $(1, 2)$ is:

   (A) $3 \cdot \binom{197}{99}$ (B) $3 \cdot \binom{100}{50}$ (C) $2 \cdot \binom{197}{98}$ (D) $3 \cdot \binom{197}{100}$.

10. Let $f(x) = \frac{1}{2} x \sin x - (1 - \cos x)$. The smallest positive integer k such that $\lim_{x \to 0} \frac{f(x)}{x^k} \neq 0$ is:

    (A) 3 (B) 4 (C) 5 (D) 6.

11. Nine students in a class gave a test for 50 marks. Let $S_1 \leq S_2 \leq \cdots \leq S_5 \leq \cdots \leq S_8 \leq S_9$ denote their ordered scores. Given that $S_1 = 20$ and $\sum_{i=1}^{9} S_i = 250$, let m be the smallest value that S_5 can take and M be the largest value that S_5 can take. Then the pair (m, M) is given by
    (A) $(20, 35)$ (B) $(20, 34)$ (C) $(25, 34)$ (D) $(25, 50)$.

12. Let 10 red balls and 10 white balls be arranged in a straight line such that 10 each are on either side of a central mark. The number of such symmetrical arrangements about the central mark is
    (A) $\frac{10!}{5!\,5!}$ (B) $10!$ (C) $\frac{10!}{5!}$ (D) $2 \cdot 10!$

13. If $z = x + iy$ is a complex number such that $\left|\frac{z-i}{z+i}\right| < 1$, then we must have

    (A) $x > 0$ (B) $x < 0$ (C) $y > 0$ (D) $y < 0$.

14. Let $S = \{x - y \mid x, y \text{ are real numbers with } x^2 + y^2 = 1\}$. Then the maximum number in the set S is

(A) 1 (B) $\sqrt{2}$ (C) $2\sqrt{2}$ (D) $1 + \sqrt{2}$.

15. In a factory, 20 workers start working on a project of packing consignments. They need exactly 5 hours to pack one consignment. Every hour 4 new workers join the existing workforce. It is mandatory to relieve a worker after 10 hours. Then the number of consignments that would be packed in the initial 113 hours is

(A) 40 (B) 50 (C) 45 (D) 52.

16. Let $ABCD$ be a rectangle with its shorter side $a > 0$ units and perimeter $2s$ units. Let $PQRS$ be any rectangle such that vertices A, B, C and D respectively lie on the lines PQ, QR, RS and SP. Then the maximum area of such a rectangle $PQRS$ in square units is given by

(A) s^2 (B) $2a\,(s-a)$ (C) $\frac{s^2}{2}$ (D) $\frac{5}{2}a\,(s-a)$.

17. The number of pairs of integers (x, y) satisfying the equation $xy(x + y + 1) = 5^{2018} + 1$ is:

(A) 0 (B) 2 (C) 1009 (D) 2018.

18. Let $p(n)$ be the number of digits when 8^n is written in base 6, and let $q(n)$ be the number of digits when 6^n is written in base 4. For example, 8^2 in base 6 is 144, hence $p(2) = 3$. Then $\lim_{n\to\infty} \frac{p(n)q(n)}{n^2}$ equals:

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) 2.

19. For a real number α, let S_α denote the set of those real numbers β that satisfy $\alpha \sin(\beta) = \beta \sin(\alpha)$. Then which of the following statements is true ?

(A) For any α, S_α is an infinite set.

(B) S_α is a finite set if and only if α is not an integer multiple of π.

(C) There are infinitely many numbers α for which S_α is the set of all real numbers.

(D) S_α is always finite.

20. If $A = \begin{pmatrix} 1 & 1 \\ 0 & i \end{pmatrix}$ and $A^{2018} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $a + d$ equals:

(A) $1 + i$ (B) 0 (C) 2 (D) 2018.

21. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions. Consider the following two statements:
P(1): If $\lim_{x\to 0} f(x)$ exists and $\lim_{x\to 0} f(x)g(x)$ exists, then $\lim_{x\to 0} g(x)$ must exist.
P(2): If f, g are differentiable with $f(x) < g(x)$ for every real number x, then $f'(x) < g'(x)$ for all x.
Then, which one of the following is a correct statement?

(A) Both P(1) and P(2) are true.
(B) Both P(1) and P(2) are false.
(C) P(1) is true and P(2) is false.
(D) P(1) is false and P(2) is true.

22. The number of solutions of the equation $\sin(7x) + \sin(3x) = 0$ with $0 \le x \le 2\pi$ is

(A) 9 (B) 12 (C) 15 (D) 18.

23. A bag contains some candies, $\frac{2}{5}$ of them are made of white chocolate and the remaining $\frac{3}{5}$ are made of dark chocolate. Out of the white chocolate candies, $\frac{1}{3}$ are wrapped in red paper, the rest are wrapped in blue paper. Out of the dark chocolate candies, $\frac{2}{3}$ are wrapped in red paper, the rest are wrapped in blue paper. If a randomly selected candy from the bag is found to be wrapped in red paper, then what is the probability that it is made up of dark chocolate?

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{3}{5}$ (D) $\frac{1}{4}$

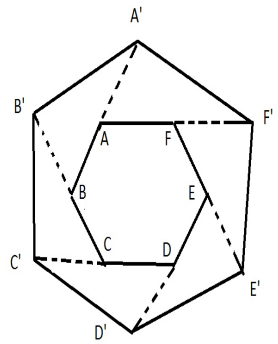
24. A party is attended by twenty people. In any subset of four people, there is at least one person who knows the other three (we assume that if X knows Y, then Y knows X). Suppose there are three people in the party who do not know each other. How many people in the party know everyone?

(A) 16 (B) 17 (C) 18
(D) Cannot be determined from the given data.

25. The sum of all natural numbers a such that $a^2 - 16a + 67$ is a perfect square is:

(A) 10 (B) 12 (C) 16 (D) 22.

26. The sides of a regular hexagon $ABCDEF$ are extended by doubling them (for example, BA extends to BA' with $BA' = 2BA$) to form a bigger regular hexagon $A'B'C'D'E'F'$ as in the figure.



Then, the ratio of the areas of the bigger to the smaller hexagon is:
(A) 2 (B) 3 (C) $2\sqrt{3}$ (D) π.

27. Between 12 noon and 1 PM, there are two instants when the hour hand and the minute hand of a clock are at right angles. The difference in minutes between these two instants is:

(A) $32\frac{8}{11}$ (B) $30\frac{8}{11}$ (C) $32\frac{5}{11}$ (D) $30\frac{5}{11}$.

28. For which values of θ, with $0 < \theta < \pi/2$, does the quadratic polynomial in t given by $t^2 + 4t\cos\theta + \cot\theta$ have repeated roots?

(A) $\frac{\pi}{6}$ or $\frac{5\pi}{18}$ (B) $\frac{\pi}{6}$ or $\frac{5\pi}{12}$ (C) $\frac{\pi}{12}$ or $\frac{5\pi}{18}$ (D) $\frac{\pi}{12}$ or $\frac{5\pi}{12}$

29. Let α, β, γ be complex numbers which are the vertices of an equilateral triangle. Then, we must have:

(A) $\alpha + \beta + \gamma = 0$ (B) $\alpha^2 + \beta^2 + \gamma^2 = 0$
(C) $\alpha^2+\beta^2+\gamma^2+\alpha\beta+\beta\gamma+\gamma\alpha = 0$ (D) $(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2 = 0$

30. Assume that n copies of unit cubes are glued together side by side to form a rectangular solid block. If the number of unit cubes that are completely invisible is 30, then the minimum possible value of n is:

(A) 204 (B) 180 (C) 140 (D) 84.